Calculus – Lesson 2: Linear, Quadratic, and Polynomial Functions (page 1)
Upward Bound Summer 2018 **Name: ____________________**

Basic: Classifying and Combining Polynomials

A **polynomial** is a finite sum of constant multiples of whole number powers of a variable.

- The **degree** of a polynomial is the highest power appearing in the polynomial.
- The **leading coefficient** is the coefficient in the highest power term.
- The number of terms determines whether it is a **monomial** (1 term), **binomial** (2 terms), **trinomial** (3 terms), or a polynomial with more than 3 terms.
- You should always simplify polynomials before determining their degree, leading coefficient, and number of terms. $x^2 - x^6 + x^6 - 1$, for example, is a quadratic (degree 2) binomial.
- When a polynomial is simplified and the terms are ordered from highest to lowest power, we say the polynomial is in **standard form** (also called **general form**).

(1) Explain why each of the following expressions is not a polynomial:

- $\sqrt[3]{x}$
- $5/x$
- 7^x
- $1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720} + \frac{x^8}{40320} - \cdots$

(2) Provide an example for each cell in the following table. If no example exists, write "impossible".

	Monomial (1 Term)	**Binomial (2 Terms)**	**Trinomial (3 Terms)**	**More than 3 Terms**
Constant (Degree 0)				
Linear (Degree 1)				
Quadratic (Degree 2)				
Cubic (Degree 3)				
Higher than 3rd Degree				

(3) What is the only polynomial which cannot fit into any cell in the above table?

To add or subtract polynomials, we simply combine "like terms" (terms with the same power on the variable). For example, $(x + 5) + (x^2 + 2x + 4) = x^2 + 3x + 9$ and $(x + 5) - (x^2 + 2x + 4) = -x^2 - x + 1$.

To multiply polynomials, we must use the generalized distributive property: every term of the first polynomial must be multiplied by every term of the second polynomial. For example, $(x + 5)(x^2 + 2x + 4)$ will be a sum of 6 products: $(x)(x^2) + (x)(2x) + (x)(4) + (5)(x^2) + (5)(2x) + (5)(4)$. This can be simplified into $x^3 + 2x^2 + 4x + 5x^2 + 10x + 20$ which further simplifies into $x^3 + 7x^2 + 14x + 20$.

(4) For each of the following pairs of polynomials, find the sum, difference, and product.

Polynomial 1	**Polynomial 2**	**Sum (P1+P2)**	**Difference (P1-P2)**	**Product (P1*P2)**
$2x+1$	$x-7$			
x^2+16	x^2-16			
x^2+5x+6	$3x$			
x^2+3x+2	$2x^2-6$			
x^2-x+1	$2x^2+3x+4$			

Intermediate: Factoring Methods

If you divide polynomials, it is possible that your result is no longer a polynomial. For example, x^2+3x+2 divided by $x+1$ is the polynomial $x+2$, but x^2+7 divided by $x+1$ is $x-1+\frac{8}{x+1}$.

Hence, we usually say the opposite of multiplying polynomials is **factoring** them. We say x^2+3x+2 factors into $(x+1)(x+2)$, but x^2+7 is **prime** (it cannot be factored). Technically we could factor x^2+7 using complex numbers, but it is prime over the set of integers, rational numbers, and real numbers.

There are various factoring methods:

- **Factoring by GCF:** If all the terms of a polynomial have a factor in common, we always start by factoring out everything they have in common. "GCF" here stands for "Greatest Common Factor". This method is also called "Factoring by Grouping".
- **Difference of Squares:** $A^2-B^2=(A+B)(A-B)$, where A and B represent any 2 expressions.
- **Difference of Cubes:** $A^3-B^3=(A-B)(A^2+AB+B^2)$
- **Sum of Cubes:** $A^3+B^3=(A+B)(A^2-AB+B^2)$
- **The "c" Method:** When a quadratic is of the form x^2+bx+c, find two numbers m and n such that $mn=c$ and $m+n=b$. Then the quadratic factors as $(x+m)(x+n)$. This can go by numerous other names, but I prefer to call it the "c" method because of the focus on finding factors of c.
- **The "ac" Method:** When a quadratic is of the form ax^2+bx+c and $a\neq 1$, find two numbers m and n such that $mn=ac$ and $m+n=b$. Split the linear term so it looks like $ax^2+mx+nx+c$. Now you should be able to factor the first two terms and the last two terms and then see the same binomial in parentheses for both resulting terms. This allows you to then combine those terms into a single factored expression. As with the "c" method, the "ac" method can go by many different names and have many minor variations (including drawing a box), but I prefer to call it the "ac" method because of the focus on finding factors of ac.
- The **Rational Roots Theorem** tells you that if a root of a polynomial is a rational number then that rational number must be of the form $\pm\frac{m}{n}$, where m is a factor of the leading coefficient and n is a factor of the constant term. You may then use **polynomial long division** or **synthetic division** to factor out the corresponding factor from the polynomial.

(5) Factor each of the following polynomials.

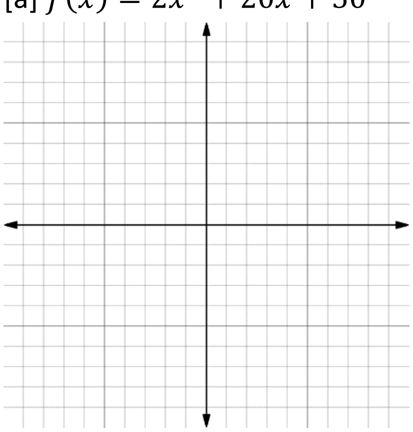
- $x^2 + 3x$
- $4x^4 - 4x^3 + x^2$
- $x^3 - 25x$
- $16x^4 - 81$
- $6x^2 - x - 15$
- $x^3 + 2x^2 - 11x - 12$

<u>Advanced: What do we know about graphing polynomials so far?</u>

You should know how to find the vertex of a parabola (quadratic), slope of a line, y-intercept of any polynomial, x-intercepts of any polynomial you can factor, and end behavior of any polynomial. Later in this course, we will learn how to find the turning points, but right now the exact coordinates of those is a mystery unless you use a graphing calculator.

(6) Graph each of the following polynomials to the best of your current ability without a graphing calculator.

[a] $f(x) = 2x^2 + 20x + 50$



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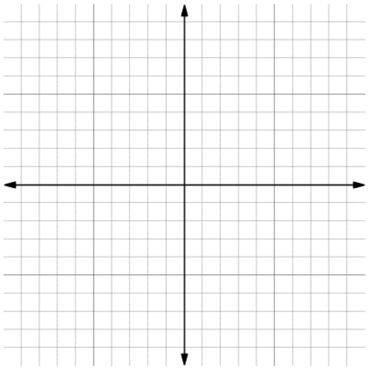
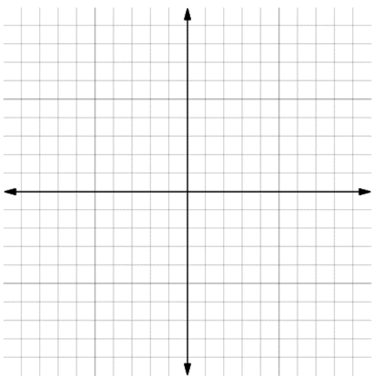
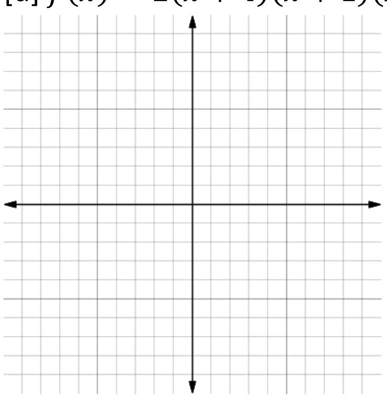
[b] $g(x) = -2(x+3)(x-3)$



[c] $h(x) = 3(x-7)^2 - 11$



[d] $f(x) = 2(x+4)(x+1)(x-3)(x-4)$



[e] $g(x) = (x+3)^3(x+2)x^2$